



# “Voice Conversion Based on Maximum Likelihood Estimation of Spectral Parameter Trajectory”

- Tomoki Toda, Alan W. Black and Keiichi Tokuda

Under the guidance of:

Dr. R. hegde

By:-

Mayank Sirotiya (Y8104036)

Vipul Arora (Y5508)

# Abstract:

- This paper describes a method for Spectral Estimation for Voice Conversion application, based on Maximum Likelihood Estimation technique.
- It also presents the conventional methods of spectral parameter conversion and addresses the problems with them.

# Introduction:

- Voice conversion technology enables a user to transform one person's speech pattern into another pattern with distinct characteristics
- A mapping function is used which consists of utterance pairs of source and target voices

# Applications

- Speaker conversion
- Cross-language speaker conversion
- Narrow-band to wide-band speech for telecommunication
- Speaking aid
- Modeling of speech production etc.

# Classical Approaches and their Limitations:

- A. Codebook mapping based on hard clustering and discrete mapping

$$\hat{\mathbf{y}}_t = \mathbf{c}_m^{(y)}$$

- B. Fuzzy vector quantization, for soft clustering

$$\hat{\mathbf{y}}_t = \sum_{m=1}^M w_{m,t}^{(x)} \mathbf{c}_m^{(y)}$$

# Classical Approaches and their Limitations:

- C. More variable representation, by modeling a difference vector

$$\hat{\mathbf{y}}_t = \mathbf{x}_t + \sum_{m=1}^M w_{m,t}^{(x)} \left( \mathbf{c}_m^{(y)} - \mathbf{c}_m^{(x)} \right)$$

- D. Method using linear multivariate regression (LMR)

$$\hat{\mathbf{y}}_t = \mathbf{A}_m \mathbf{x}_t + \mathbf{b}_m$$

# Classical Approaches and their Limitations:

## E. Gaussian mixture model

It realizes continuous mapping based on soft clustering

$$\hat{\mathbf{y}}_t = \sum_{m=1}^M w_{m,t}^{(x)} (\mathbf{A}_m \mathbf{x}_t + \mathbf{b}_m)$$

# Conventional GMM based mapping

- The joint probability density of the source and target feature vectors is

$$P(z_t | \lambda^{(z)}) = \sum_{m=1}^M w_m N(z_t; \mu_m^{(z)}, \Sigma_m^{(z)})$$

where  $z_t$  is a joint vector  $[\mathbf{x}_t^T, \mathbf{y}_t^T]^T$   
and the mean vector and covariance matrix are written as

$$\mu_m^{(z)} = \begin{bmatrix} \mu_m^{(x)} \\ \mu_m^{(y)} \end{bmatrix}, \Sigma_m^{(z)} = \begin{bmatrix} \Sigma_m^{(xx)} & \Sigma_m^{(xy)} \\ \Sigma_m^{(yx)} & \Sigma_m^{(yy)} \end{bmatrix}$$



# Conventional GMM based mapping

- Conditional probability density can also be represented as

$$P(y_t | \mathbf{x}_t, \lambda^{(z)}) = \sum_{m=1}^M P(m | \mathbf{x}_t, \lambda^{(z)}) P(y_t | \mathbf{x}_t, m, \lambda^{(z)})$$

where

$$P(m | \mathbf{x}_t, \lambda^{(z)}) = \frac{w_m N(\mathbf{x}_t; \mu_m^{(x)}, \Sigma_m^{(xx)})}{\sum_{n=1}^M w_n N(\mathbf{x}_t; \mu_n^{(x)}, \Sigma_n^{(xx)})}$$

$$P(y_t | \mathbf{x}_t, m, \lambda^{(z)}) = N(y_t; E_{m,t}^{(y)}, D_m^{(y)})$$

# Conventional GMM based mapping

- The mean vector and the covariance matrix of m'th conditional probability distribution are written as

$$E_{m,t}^{(y)} = \mu_m^{(y)} + \sum_m^{(yx)} \sum_m^{(xx)-1} (x_t - \mu_m^{(x)})$$

$$D_m^{(y)} = \sum_m^{(yy)} - \sum_m^{(yx)} \sum_m^{(xx)-1} \sum_m^{(xy)}$$

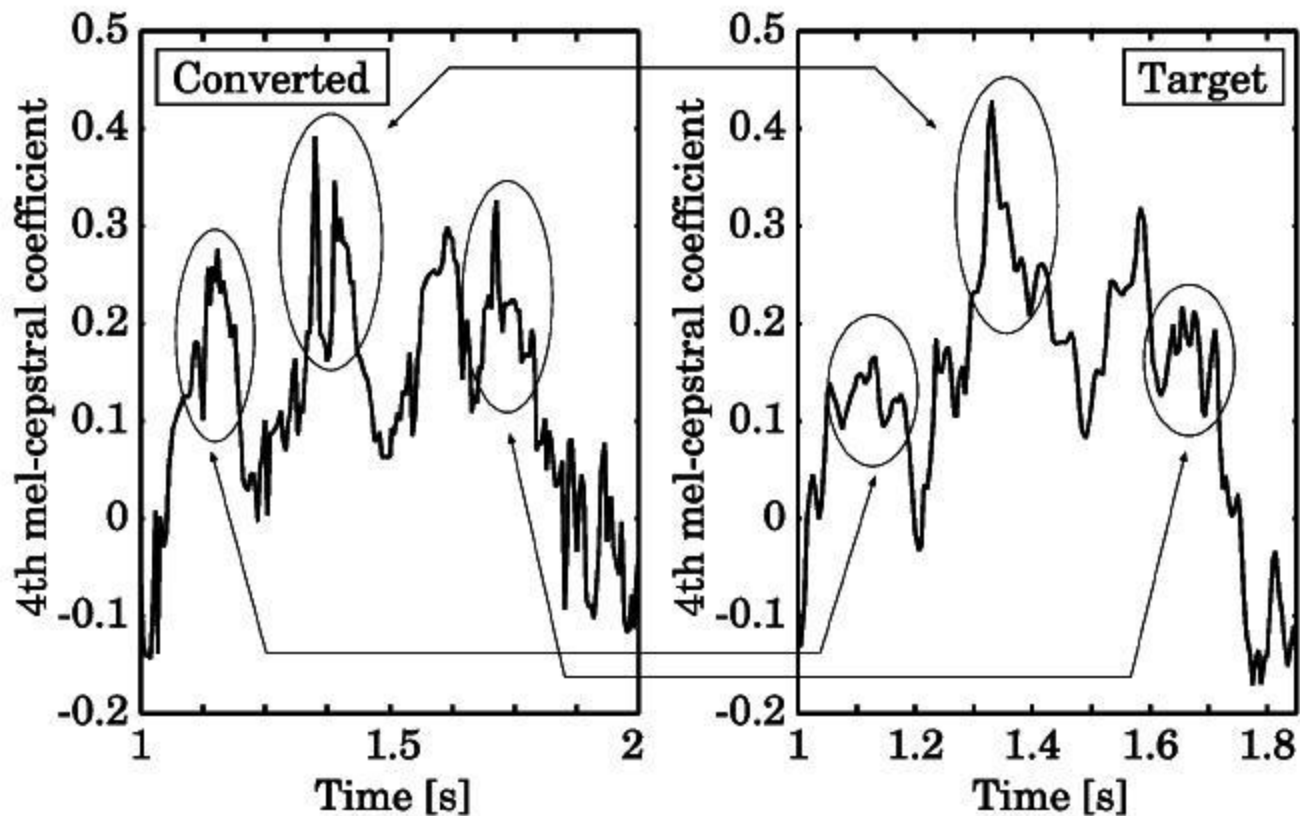
# Conventional GMM based mapping

- In conventional method the conversion is based on MMSE as follows

$$\begin{aligned}\hat{\mathbf{y}}_t &= E[\mathbf{y}_t|\mathbf{x}_t] \\ &= \int P(\mathbf{y}_t|\mathbf{x}_t, \boldsymbol{\lambda}^{(z)}) \mathbf{y}_t d\mathbf{y}_t \\ &= \int \sum_{m=1}^M P(m|\mathbf{x}_t, \boldsymbol{\lambda}^{(z)}) P(\mathbf{y}_t|\mathbf{x}_t, m, \boldsymbol{\lambda}^{(z)}) \mathbf{y}_t d\mathbf{y}_t \\ &= \sum_{m=1}^M P(m|\mathbf{x}_t, \boldsymbol{\lambda}^{(z)}) E_{m,t}^{(y)}\end{aligned}$$

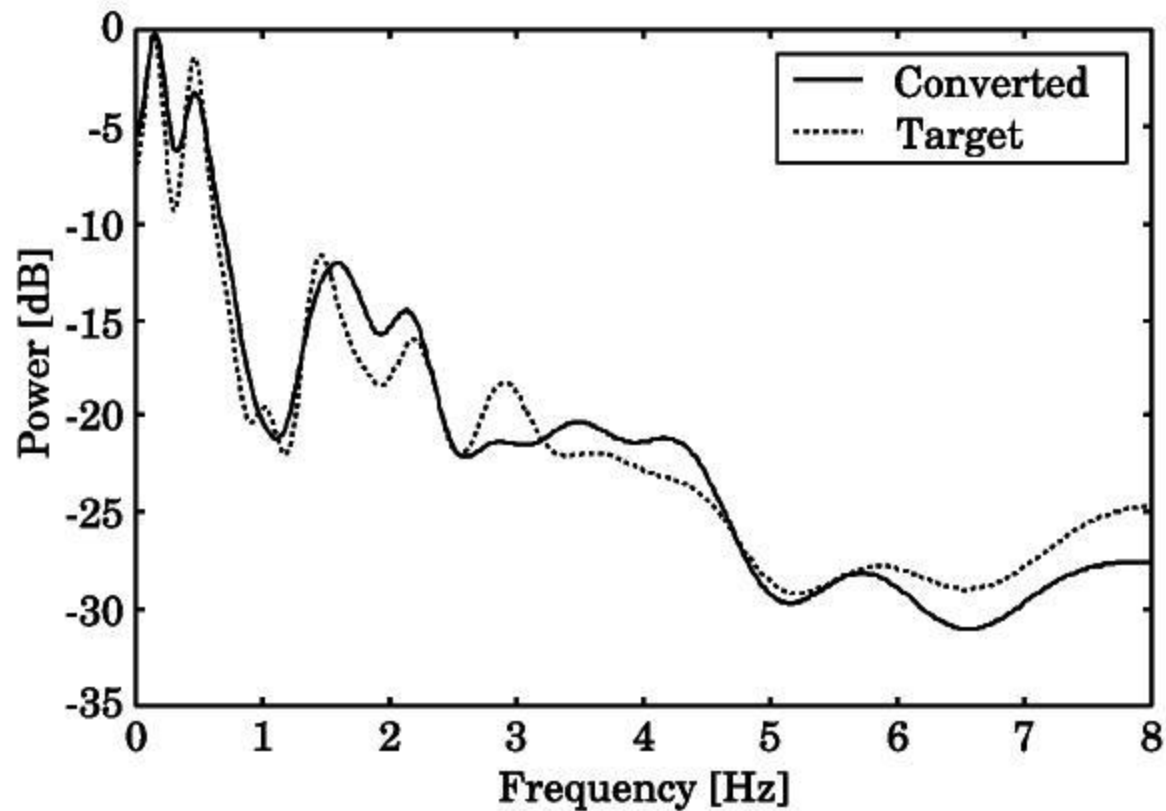
# Drawbacks

## I. Time-independent mapping



# Drawbacks

## 2. Oversmoothing





# PROPOSED SPECTRAL CONVERSION

- Trajectory based spectral conversion process, instead of conventional frame based one.

$$\hat{\mathbf{y}} = f(\mathbf{x})$$

where,

$$\mathbf{x} = [\mathbf{x}_1^\top, \mathbf{x}_2^\top, \dots, \mathbf{x}_t^\top, \dots, \mathbf{x}_T^\top]^\top$$

$$\mathbf{y} = [\mathbf{y}_1^\top, \mathbf{y}_2^\top, \dots, \mathbf{y}_t^\top, \dots, \mathbf{y}_T^\top]^\top$$

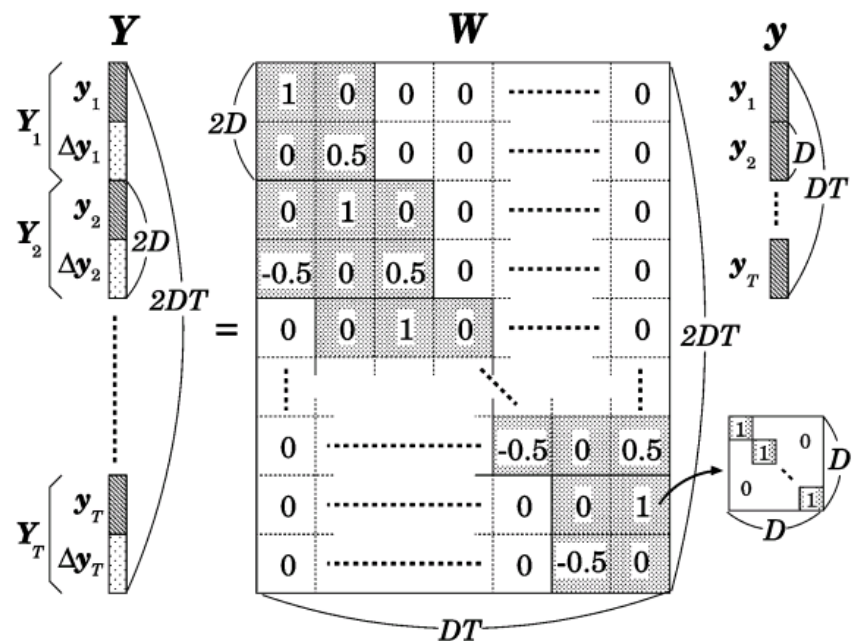
- Conversion considering feature correlation between frames (dynamic features)

$$X = \left[ X_1^\top, X_2^\top, \dots, X_t^\top, \dots, X_T^\top \right]^\top$$

$$Y = \left[ Y_1^\top, Y_2^\top, \dots, Y_t^\top, \dots, Y_T^\top \right]^\top$$

$$X_t = \left[ \mathbf{x}_t^\top, \Delta \mathbf{x}_t^\top \right]^\top \text{ and } Y_t = \left[ \mathbf{y}_t^\top, \Delta \mathbf{y}_t^\top \right]^\top$$

$$Y = W y$$





# MLE of Parameter Trajectory

- The joint vector is

$$\mathbf{Z}_t = \left[ \mathbf{X}_t^\top, \mathbf{Y}_t^\top \right]^\top$$

- The GMM of joint probability density

$$P(\mathbf{Z}_t | \boldsymbol{\lambda}^{(Z)})$$

is trained using conventional training framework.

- Likelihood Function to be maximized:

$$P(\mathbf{Y} | \mathbf{X}, \boldsymbol{\lambda}^{(Z)}) = \prod_{t=1}^T \sum_{m=1}^M P(m | \mathbf{X}_t, \boldsymbol{\lambda}^{(Z)}) \times P(\mathbf{Y}_t | \mathbf{X}_t, m, \boldsymbol{\lambda}^{(Z)})$$

where,

$$P\left(m|X_t, \boldsymbol{\lambda}^{(Z)}\right) = \frac{w_m \mathcal{N}\left(X_t; \boldsymbol{\mu}_m^{(X)}, \boldsymbol{\Sigma}_m^{(XX)}\right)}{\sum_{n=1}^M w_n \mathcal{N}\left(X_t; \boldsymbol{\mu}_n^{(X)}, \boldsymbol{\Sigma}_n^{(XX)}\right)}$$

$$P\left(Y_t|X_t, m, \boldsymbol{\lambda}^{(Z)}\right) = \mathcal{N}\left(Y_t; \boldsymbol{E}_{m,t}^{(Y)}, \boldsymbol{D}_m^{(Y)}\right)$$

here,

$$\boldsymbol{E}_{m,t}^{(Y)} = \boldsymbol{\mu}_m^{(Y)} + \boldsymbol{\Sigma}_m^{(YX)} \boldsymbol{\Sigma}_m^{(XX)^{-1}} \left(X_t - \boldsymbol{\mu}_m^{(X)}\right)$$

$$\boldsymbol{D}_m^{(Y)} = \boldsymbol{\Sigma}_m^{(YY)} - \boldsymbol{\Sigma}_m^{(YX)} \boldsymbol{\Sigma}_m^{(XX)^{-1}} \boldsymbol{\Sigma}_m^{(XY)}$$

# Derivation of Conditional Probability

$$\mathbf{x} = \begin{pmatrix} \mathbf{x}_a \\ \mathbf{x}_b \end{pmatrix} \quad \boldsymbol{\mu} = \begin{pmatrix} \boldsymbol{\mu}_a \\ \boldsymbol{\mu}_b \end{pmatrix} \quad \boldsymbol{\Sigma} = \begin{pmatrix} \boldsymbol{\Sigma}_{aa} & \boldsymbol{\Sigma}_{ab} \\ \boldsymbol{\Sigma}_{ba} & \boldsymbol{\Sigma}_{bb} \end{pmatrix}$$

$$\boldsymbol{\Lambda} \equiv \boldsymbol{\Sigma}^{-1} \quad \boldsymbol{\Lambda} = \begin{pmatrix} \boldsymbol{\Lambda}_{aa} & \boldsymbol{\Lambda}_{ab} \\ \boldsymbol{\Lambda}_{ba} & \boldsymbol{\Lambda}_{bb} \end{pmatrix}$$

$$\begin{aligned} -\frac{1}{2}(\mathbf{x} - \boldsymbol{\mu})^T \boldsymbol{\Sigma}^{-1}(\mathbf{x} - \boldsymbol{\mu}) = \\ -\frac{1}{2}(\mathbf{x}_a - \boldsymbol{\mu}_a)^T \boldsymbol{\Lambda}_{aa}(\mathbf{x}_a - \boldsymbol{\mu}_a) - \frac{1}{2}(\mathbf{x}_a - \boldsymbol{\mu}_a)^T \boldsymbol{\Lambda}_{ab}(\mathbf{x}_b - \boldsymbol{\mu}_b) \\ - \frac{1}{2}(\mathbf{x}_b - \boldsymbol{\mu}_b)^T \boldsymbol{\Lambda}_{ba}(\mathbf{x}_a - \boldsymbol{\mu}_a) - \frac{1}{2}(\mathbf{x}_b - \boldsymbol{\mu}_b)^T \boldsymbol{\Lambda}_{bb}(\mathbf{x}_b - \boldsymbol{\mu}_b) \end{aligned}$$

# Derivation of Conditional Probability

$$-\frac{1}{2}(\mathbf{x} - \boldsymbol{\mu})^T \boldsymbol{\Sigma}^{-1}(\mathbf{x} - \boldsymbol{\mu}) = -\frac{1}{2}\mathbf{x}^T \boldsymbol{\Sigma}^{-1}\mathbf{x} + \mathbf{x}^T \boldsymbol{\Sigma}^{-1}\boldsymbol{\mu} + \text{const}$$

- Second order term is,

$$-\frac{1}{2}\mathbf{x}_a^T \boldsymbol{\Lambda}_{aa}\mathbf{x}_a$$

- First order term is,

$$\mathbf{x}_a^T \{ \boldsymbol{\Lambda}_{aa}\boldsymbol{\mu}_a - \boldsymbol{\Lambda}_{ab}(\mathbf{x}_b - \boldsymbol{\mu}_b) \}$$

## i) EM Algorithm

$$\hat{\mathbf{y}} = \arg \max P \left( \mathbf{Y} | \mathbf{X}, \boldsymbol{\lambda}^{(Z)} \right)$$

$$\begin{aligned} Q(\mathbf{Y}, \hat{\mathbf{Y}}) &= \sum_{\text{all } \mathbf{m}} P \left( \mathbf{m} | \mathbf{X}, \mathbf{Y}, \boldsymbol{\lambda}^{(Z)} \right) \log P \left( \hat{\mathbf{Y}}, \mathbf{m} | \mathbf{X}, \boldsymbol{\lambda}^{(Z)} \right) \\ &= \sum_{t=1}^T \sum_{m=1}^M P \left( m | \mathbf{X}_t, \mathbf{Y}_t, \boldsymbol{\lambda}^{(Z)} \right) \log P \left( \hat{\mathbf{Y}}_t, m | \mathbf{X}_t, \boldsymbol{\lambda}^{(Z)} \right) \\ &= \sum_{t=1}^T \sum_{m=1}^M \gamma_{m,t} \left( -\frac{1}{2} \hat{\mathbf{Y}}_t^\top \mathbf{D}_m^{(Y)}{}^{-1} \hat{\mathbf{Y}}_t + \hat{\mathbf{Y}}_t^\top \mathbf{D}_m^{(Y)}{}^{-1} \mathbf{E}_{m,t}^{(Y)} \right) + \overline{K} \end{aligned}$$

$$\begin{aligned}
Q(Y, \hat{Y}) &= \sum_{t=1}^T -\frac{1}{2} \hat{Y}_t^\top \overline{D_t^{(Y)}^{-1}} \hat{Y}_t + \hat{Y}_t^\top \overline{D_t^{(Y)}^{-1} E_t^{(Y)}} + \bar{K} \\
&= -\frac{1}{2} \hat{Y}^\top \overline{D^{(Y)}^{-1}} \hat{Y} + \hat{Y}^\top \overline{D^{(Y)}^{-1} E^{(Y)}} + \bar{K} \\
&= -\frac{1}{2} \hat{\mathbf{y}}^\top \mathbf{W}^\top \overline{D^{(Y)}^{-1}} \mathbf{W} \hat{\mathbf{y}} + \hat{\mathbf{y}}^\top \mathbf{W}^\top \overline{D^{(Y)}^{-1} E^{(Y)}} + \bar{K}
\end{aligned}$$

$$\frac{\partial Q(Y, \hat{Y})}{\partial \mathbf{y}} = -\mathbf{W}^\top \overline{D_{\hat{\mathbf{m}}}^{(Y)}^{-1}} \mathbf{W} \mathbf{y} + \mathbf{W}^\top \overline{D_{\hat{\mathbf{m}}}^{(Y)}^{-1} E_{\hat{\mathbf{m}}}^{(Y)}}$$

- Equating it to zero, we get

$$\hat{\mathbf{y}} = \left( \mathbf{W}^\top \overline{D^{(Y)}^{-1}} \mathbf{W} \right)^{-1} \mathbf{W}^\top \overline{D^{(Y)}^{-1} E^{(Y)}}$$

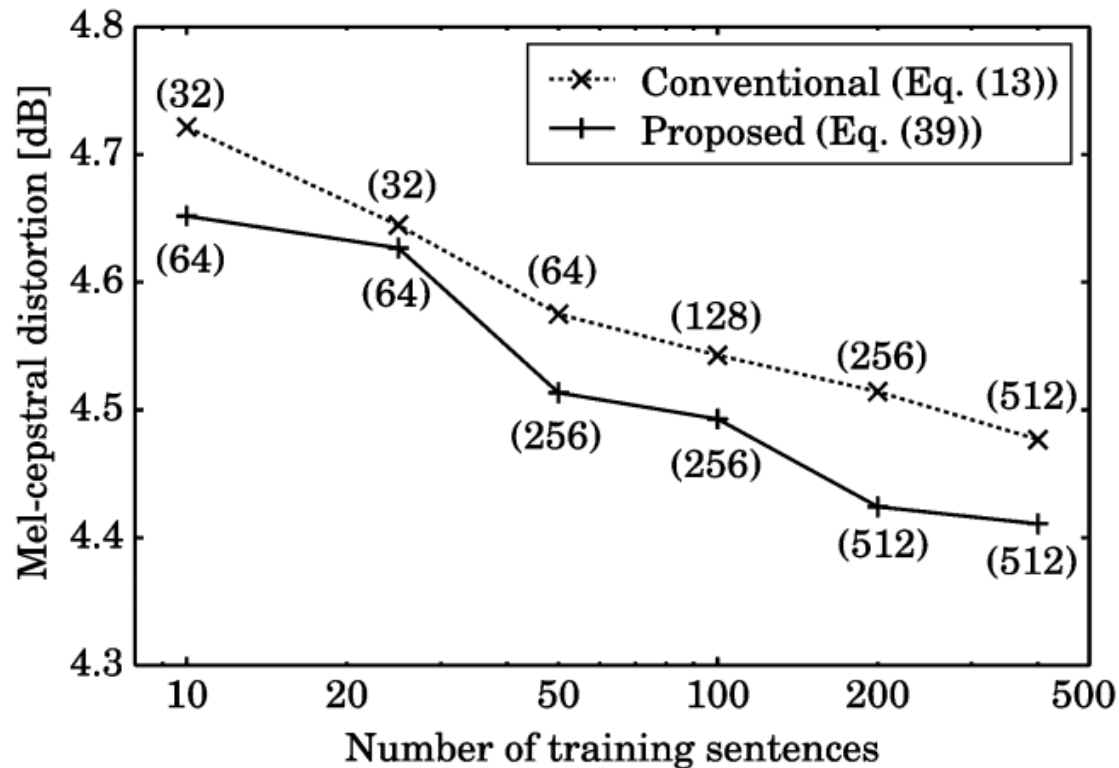
## ii) Approximation with suboptimum mixture sequence

$$\hat{\boldsymbol{m}} = \operatorname{argmax} P \left( \boldsymbol{m} | \boldsymbol{X}, \boldsymbol{\lambda}^{(Z)} \right)$$

$$\mathcal{L} = \log P \left( \hat{\boldsymbol{m}} | \boldsymbol{X}, \boldsymbol{\lambda}^{(Z)} \right) P \left( \boldsymbol{Y} | \boldsymbol{X}, \hat{\boldsymbol{m}}, \boldsymbol{\lambda}^{(Z)} \right)$$

$$\hat{\boldsymbol{y}} = \left( \boldsymbol{W}^\top \boldsymbol{D}_{\hat{\boldsymbol{m}}}^{(Y)-1} \boldsymbol{W} \right)^{-1} \boldsymbol{W}^\top \boldsymbol{D}_{\hat{\boldsymbol{m}}}^{(Y)-1} \boldsymbol{E}_{\hat{\boldsymbol{m}}}^{(Y)}$$

# Results:



(Mel Cepstral distortion before conversion is 7.30 dB)



# Summary

- GMM based Feature mapping
- Conditional Gaussian Distributions
- Maximum Likelihood technique for GMMs
- Expectation Maximization Algorithm